Exercise 1.1.4

Show that $|\mathbf{v}|$ is the distance of (v^1, v^2, v^3) from $(0, 0, 0)$ by two applications of the Pythagorean theorem.

Solution

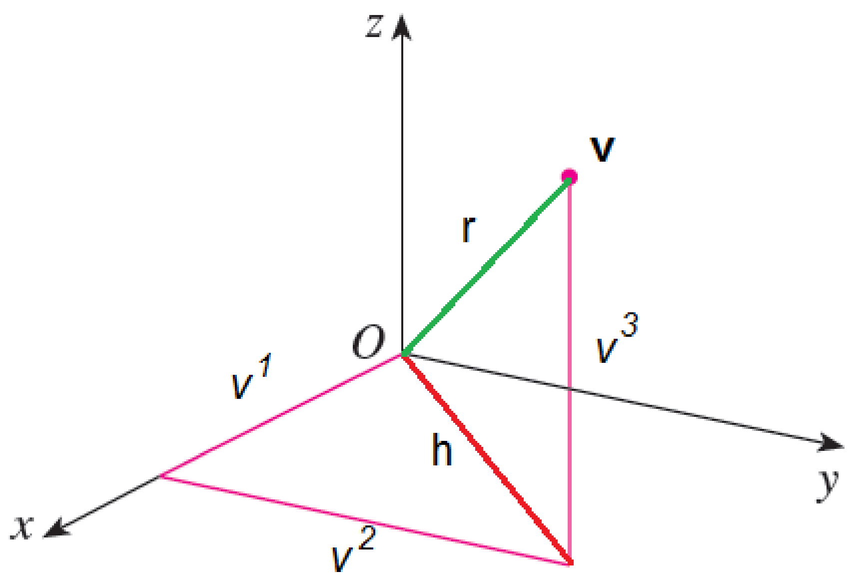

Figure 1: Arbitrary three-dimensional vector $\mathbf{v}$ in $\mathbb{R}^3$ with components v^1, v^2, and v^3.

Applying the Pythagorean theorem, h can be determined.

$$h^2 = (v^1)^2 + (v^2)^2 \tag{1}$$

Applying the Pythagorean theorem for the second time, r can be determined.

$$r^2 = h^2 + (v^3)^2$$

Substituting $|\mathbf{v}|$ for r and (1) for h^2, we get

$$\begin{aligned} |\mathbf{v}|^2 &= (v^1)^2 + (v^2)^2 + (v^3)^2 \\ |\mathbf{v}| &= \sqrt{(v^1)^2 + (v^2)^2 + (v^3)^2}. \end{aligned}$$

Therefore, $|\mathbf{v}|$ is the distance of (v^1, v^2, v^3) from $(0, 0, 0)$.